

Superconducting decay length in a ferromagnetic metal

D. Yu. Gusakova* and M. Yu. Kupriyanov

Nuclear Physics Institute, Moscow State University, Moscow, 119992 Russia

A. A. Golubov

Faculty of Science and Technology, University of Twente, 7500 AE Enschede, The Netherlands

The complex decay length ξ characterizing penetration of superconducting correlations into a ferromagnet due to the proximity effect is studied theoretically in the frame of the linearized Eilenberger equations. The real part ξ_1 and imaginary part ξ_2 of the decay length are calculated as functions of exchange energy and the rates of ordinary, spin flip and spin orbit electronic scattering in a ferromagnet. The lengths $\xi_{1,2}$ determine the spatial scales of, respectively, decay and oscillation of a critical current in SFS Josephson junctions in the limit of large distance between superconducting electrodes. The developed theory provides the criteria of applicability of the expressions for ξ_1 and ξ_2 in the dirty and the clean limits which are commonly used in the analysis of SF hybrid structures.

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The decay length ξ is an important material parameter which characterizes the scale of penetration of superconducting correlation into a non-superconducting material across an interface with a superconductor. The critical current I_C in a Josephson junction scales exponentially with the distance between the superconducting electrodes L if L is larger than ξ : $I_C \propto \exp\{-L/\xi\}$. In nonmagnetic materials the decay length is a real number, while in ferromagnets ξ is a complex number (see [1]-[4] for the reviews). In particular, if the condition of so-called dirty limit is fulfilled in the F metal, the decay length is

$$\xi^{-1} = \xi_1^{-1} + i\xi_2^{-1}, \quad \xi_{1,2}^{-1} = \sqrt{\frac{(\pi T)^2 + H^2 \pm \pi T}{D_F}}, \quad (1)$$

where D_F and H are the diffusive coefficient and the exchange field in a ferromagnet, respectively. In the clean limit

$$\xi_1^{-1} = \xi_0^{-1} + \ell^{-1}, \quad \xi_0^{-1} = \frac{2\pi T}{v_F}, \quad \xi_2^{-1} = \xi_H^{-1} = \frac{2H}{v_F}, \quad (2)$$

where v_F is the Fermi velocity in a ferromagnet and ℓ is the electron mean free path. From (1), (2) it is clearly seen that for dirty materials $\xi_2 > \xi_1$, and in the limit of large $H \gg \pi T$, the characteristic lengths are nearly equal $\xi_1 \approx \xi_2$. In the clean limit these length scales ξ_1 and ξ_2 are completely independent.

The existing experimental data obtained up to now in SFS Josephson junctions[5]-[16] can be separated into two groups depending on whether weak or strong ferromagnet was used for junction fabrication. To be considered as a weak ferromagnet, the dilute ferromagnetic alloys (e.g. $\text{Cu}_{1-x}\text{Ni}_x$) should be in the range of concentration close to the critical one ($x \approx 0,5$). The electron

mean free path in these alloys is very small providing the fulfillment of the dirty limit conditions. As a result, the observed relation between the decay (ξ_1) and oscillation (ξ_2) lengths $\xi_2 \gtrsim \xi_1$, is close to that following from (1). It is necessary to point out that in some experiments [12] the observed difference between ξ_2 and ξ_1 is so large that it can not be explained by temperature factor in (1) only and spin-dependent scattering processes should be taken into account [12],[17].

Contrary to that, in the structures with strong ferromagnet[11], [16] (Ni, Ni_3Al), the relation between ξ_1 and ξ_2 is just the opposite and large ratio $\xi_1/\xi_2 \sim 10$ was observed in Ni_3Al [16]. Therefore more complex model should be developed for the data interpretation.

Most of previous theoretical work on SF hybrids was performed assuming the dirty limit (see [2]-[4]), and only first order corrections to the decay length in small parameter $\ell\xi_H \ll 1$ were discussed in [18, 19]. Properties of SF structures in the clean limit were also studied in a number of papers, see e.g. [21, 22, 23, 24]. The purpose of this work is to develop general theory describing the decay length ξ in a ferromagnet for any relation between ξ_0 , ξ_H and ℓ .

To do this we consider a generic SFS Josephson junction with arbitrary transparency of SF interfaces and large thickness of the F layer $L \gg \xi_1$. It is well known[1]-[4] that the critical current of this structure should fall exponentially with L

$$I_C = I_0 \exp\{-L/\xi\}.$$

Here the prefactor I_0 depends on physical properties of SF interfaces and the nearby S and F regions, while ξ depends only on bulk parameters of F material and can be obtained[26],[27] as the solution of linearized quasi-classical Eilenberger equations [25]. These equations are valid at the distances from the interfaces larger than ξ and have the form [25],[3],[4]

*Electronic address: dariamessage@yandex.ru

$$\begin{aligned}
& (\xi_0^{-1} \pm i\xi_H^{-1})f_{\pm} + \cos\theta \frac{\partial}{\partial x} f_{\pm} = \\
& = \ell_{eff}^{-1} (\langle f_{\pm} \rangle - f_{\pm}) + \ell_{soeff}^{-1} (f_{\mp} - f_{\pm}), \quad (3)
\end{aligned}$$

$$\ell_{eff}^{-1} = \ell^{-1} + \ell_z^{-1} + 2\ell_x^{-1}, \quad (4)$$

$$\ell_{soeff}^{-1} = \ell_{so}^{-1} - \ell_x^{-1}, \quad \langle \dots \rangle = \int_0^\pi (\dots) \sin\theta d\theta. \quad (5)$$

Here θ is the angle between the direction of electron velocity v_F and the x -axis, which is oriented perpendicular to the interfaces, $f_{\pm} = f_{\pm}(x, \theta)$ are the quasiclassical Eilenberger functions describing the behavior of spin up and spin down electrons in the presence of exchange field H oriented parallel to the SF interfaces. The parameters $\ell_{so} = v_F \tau_{so}$, $\ell_z = v_F \tau_z$, $\ell_x = v_F \tau_x$, are the electron mean free paths for parallel and perpendicular to the direction of H magnetic scattering, while $\ell_{so} = v_F \tau_{so}$ is the electron mean free path for spin orbit interaction.

Solution of Eq.(3) has the form

$$f_{\pm}(x, \theta) = C_{\pm}(\theta) \exp\left\{-\frac{x}{\xi}\right\}, \quad \xi^{-1} = \xi_1^{-1} + i\xi_2^{-1} \quad (6)$$

where ξ is the effective decay length independent on θ . Substitution of (6) into (3) provides the system of two equations for $C_{\pm}(\theta)$

$$\begin{aligned}
& (\xi_0^{-1} + i\xi_H^{-1})C_+(\theta) - \xi^{-1} \cos\theta C_+(\theta) = \\
& = \ell_{eff}^{-1} (\langle C_+(\theta) \rangle - C_+(\theta)) + \ell_{soeff}^{-1} (C_-(\theta) - C_+(\theta)) \quad (7)
\end{aligned}$$

$$\begin{aligned}
& (\xi_0^{-1} - i\xi_H^{-1})C_-(\theta) - \xi^{-1} \cos\theta C_-(\theta) = \\
& = \ell_{eff}^{-1} (\langle C_-(\theta) \rangle - C_-(\theta)) + \ell_{soeff}^{-1} (C_+(\theta) - C_-(\theta)) \quad (8)
\end{aligned}$$

Solution of these equations has the form

$$C_+(\theta) = \frac{\langle C_+(\theta) \rangle \Lambda_+^{-1} + \ell_{soeff}^{-1} \langle C_-(\theta) \rangle}{\ell_{eff} \left(\Lambda_+^{-1} \Lambda_-^{-1} - \ell_{soeff}^{-2} \right)} \quad (9)$$

$$C_-(\theta) = \frac{\langle C_-(\theta) \rangle \Lambda_+^{-1} + \ell_{soeff}^{-1} \langle C_+(\theta) \rangle}{\ell_{eff} \left(\Lambda_+^{-1} \Lambda_-^{-1} - \ell_{soeff}^{-2} \right)} \quad (10)$$

$$\xi_{10}^{-1} = \xi_0^{-1} + \ell_{eff}^{-1} + \ell_{soeff}^{-1}$$

$$\Lambda_{\pm}^{-1} = \xi_{10}^{-1} - \xi^{-1} \cos\theta \pm i\xi_H^{-1}$$

Averaging in (9), (10) over angle θ we get the system of two equations for $\langle C_{\pm}(\theta) \rangle$. Its compatibility condition results in the equation for the effective decay length ξ_{eff}

$$\tanh \frac{\xi^{-1}}{\ell_{eff}^{-1}} = \frac{\xi^{-1}}{\xi_{10}^{-1} \pm \sqrt{(\ell_{soeff}^{-2} - \xi_H^{-2})}}. \quad (11)$$

It is clearly seen that if the effective spin orbit interaction is so strong that $\ell_{soeff}^{-1} \geq \xi_H^{-1}$, then the right hand side of (11) is real. Therefore in this case Eq. (11) provides us by two solutions for ξ_1^{-1} , while $\xi_2^{-1} = 0$. It is necessary to mention that in the absence of ferromagnetic ordering ($H = 0$) due to degeneracy in spin orientation the critical current must not depend on ℓ_{soeff} . In this situation only the root of equation corresponding to the '+' sign in Eq. (11) should be considered

$$\tanh \frac{\xi_{11}^{-1}}{\ell_{eff}^{-1}} = \frac{\xi_{11}^{-1}}{\xi_0^{-1} + \ell_{eff}^{-1}} \quad (12)$$

which provides the largest value of the decay length.

Solution of Eq. (11)

$$\tanh \frac{\xi_{12}^{-1}}{\ell_{eff}^{-1}} = \frac{\xi_{12}^{-1}}{\xi_0^{-1} + \ell_{eff}^{-1} + 2\ell_{soeff}^{-1}} \quad (13)$$

with the smaller $\xi = \xi_{12}$ also exists at finite H . (In the limit $H \rightarrow 0$ the prefactor before this exponential solutions goes to zero [17] providing independence of the critical current on ξ_{12}). At $\ell_{soeff} = \xi_H$ these two lengths, are equal to each other, $\xi_{11} = \xi_{12}$. With further H increase the right hand side of Eq.(11) becomes complex and Eq.(11) can be rewritten as

$$\tanh \frac{\xi^{-1}}{\ell_{eff}^{-1}} = \frac{\xi^{-1}}{\xi_{10}^{-1} + i\xi_{20}^{-1}}, \quad \xi_{20}^{-1} = \sqrt{\xi_H^{-2} - \ell_{soeff}^{-2}}. \quad (14)$$

The sign '-' in Eq.(11) simply provides the equation for the complex-conjugate solution of Eq.(14).

In the limit $\ell_{eff} \ll \xi$ one can expand the hyperbolic tangent in (14) in series keeping three first terms and get

$$\frac{\ell_{eff}}{\xi_1} = \sqrt{\frac{3\Gamma_+}{2}} \left[1 + \frac{1}{10} \left(\frac{\ell_{eff}}{\xi_{01}} - 1 - \frac{\ell_{eff}^2}{\xi_{20}^2 \Gamma_+} \right) \right], \quad (15)$$

$$\frac{\ell_{eff}}{\xi_2} = \sqrt{\frac{3\Gamma_-}{2}} \left[1 + \frac{1}{10} \left(\frac{\ell_{eff}}{\xi_{01}} - 1 + \Gamma_+ \right) \right], \quad (16)$$

$$\Gamma_{\pm} = \sqrt{\left(\frac{\ell_{eff}}{\xi_{01}} - 1 \right)^2 + \frac{\ell_{eff}^2}{\xi_{20}^2}} \pm \left(\frac{\ell_{eff}}{\xi_{01}} - 1 \right).$$

The expressions in the square brackets in (15), (16) give first order corrections to the dirty limit formula [17] for ξ_1 and ξ_2 . This approximation valid if

$$\sqrt{\xi_0^{-2} + 2\xi_0^{-1}\ell_{soeff}^{-1} + \xi_H^{-2}} \pm (\xi_0^{-1} + \ell_{soeff}^{-1}) \ll \ell_{eff}^{-1}. \quad (17)$$

In the limit $\xi_0, \ell_{soeff} \gg \xi_H$ the expression $\xi = \sqrt{\frac{D_F}{iH} (1 - \frac{2}{5}iH\tau)}$, $\tau = l/v_F$, follows from Eqs.(15), (16). This formula was obtained before in Ref. [18, 19, 20] and can be interpreted as a complex correction to the diffusion coefficient, $D_F^{eff} = D_F (1 - \frac{2}{5}iH\tau)$.

In the clean limit

$$A \gg \max \left\{ \ln \sqrt{A}, \ln \sqrt{\frac{\ell_{eff}^2}{\xi_H^2} - \frac{\ell_{eff}^2}{\ell_{soeff}^2}} \right\} \quad (18)$$

$$A = 1 + \frac{\ell_{eff}}{\xi_0} + \frac{\ell_{eff}}{\ell_{soeff}}$$

in the first approximation we may put the hyperbolic tangent in (14) equal to unity and get

$$\xi_1^{-1} = \xi_{10}^{-1}, \quad \xi_2^{-1} = \xi_{20}^{-1} \quad (19)$$

It is clearly seen that for $\ell_{soeff}^{-1} \rightarrow 0$ this formula transforms into Eq.(2). In the next approximation it is easy to get that the corrections to (19)

$$\xi_1^{-1} = \xi_{10}^{-1} - 2p \exp\left(-\frac{2\ell_{eff}}{\xi_{10}}\right) \quad (20)$$

$$\xi_2^{-1} = \xi_{20}^{-1} + 2q \exp\left(-\frac{2\ell_{eff}}{\xi_{10}}\right) \quad (21)$$

where

$$p = \xi_{20}^{-1} \sin\left(\frac{2\ell_{eff}}{\xi_{20}}\right) + \xi_{10}^{-1} \cos\left(\frac{2\ell_{eff}}{\xi_{20}}\right)$$

$$q = \xi_{10}^{-1} \sin\left(\frac{2\ell_{eff}}{\xi_{20}}\right) - \xi_{20}^{-1} \cos\left(\frac{2\ell_{eff}}{\xi_{20}}\right)$$

are oscillating functions of ξ_H .

Eq.(14) is equivalent to the system of equations for ξ_1 and ξ_2

$$\frac{\xi_1^{-1}}{\xi_{10}^{-1}} = \coth \frac{2\xi_1^{-1}}{\ell_{eff}^{-1}} - a \cos\left(\frac{2\xi_2^{-1}}{\ell_{eff}^{-1}} - \arctan \frac{\xi_2^{-1}}{\xi_{10}^{-1}}\right) \quad (22)$$

$$\frac{\xi_2^{-1}}{\xi_{20}^{-1}} = \coth \frac{2\xi_1^{-1}}{\ell_{eff}^{-1}} - b \cos\left(\frac{2\xi_2^{-1}}{\ell_{eff}^{-1}} + \arctan \frac{\xi_1^{-1}}{\xi_{20}^{-1}}\right) \quad (23)$$

$$a = \frac{\sqrt{\xi_{10}^{-2} + \xi_2^{-2}}}{\xi_{10}^{-1} \sinh \frac{2\xi_1^{-1}}{\ell_{eff}^{-1}}}, \quad b = \frac{\sqrt{\xi_{20}^{-2} + \xi_1^{-2}}}{\xi_{20}^{-1} \sinh \frac{2\xi_1^{-1}}{\ell_{eff}^{-1}}}$$

From the structure of equations (22), (23) it follows that increase of ξ_{20}^{-1} leads to increase of ξ_2^{-1} . This, in turn, results in increase of the second negative item in right hand side of (22). Since ξ_1^{-1} must be a positive value increase of ξ_2^{-1} should be accompanied by a jump, at a certain point, to the positive branch of $\cos(x)$ leading to a discontinuity of $\xi_2^{-1}(\xi_{20}^{-1})$ dependence. This consideration is proved by numerical solution of (14) (see Figs.1-3).

Figures 1 and 2 show the dependencies of $\xi_1^{-1}(\xi_{20}^{-1})$ and $\xi_2^{-1}(\xi_{20}^{-1}) - \xi_{20}^{-1}$ calculated for fixed values of parameter ξ_{10}^{-1} . Open triangles and circles in the figures show the asymptotic dependencies (15), (16) and (20), (21), respectively. It is clearly seen that in the parameter intervals $\xi_{20}^{-1} \leq 10\ell_{eff}^{-1}$, $\xi_{10}^{-1} \geq 2\ell_{eff}^{-1}$, the expressions (20), (21) provide a good fit to the exact solution of equation (14). The dirty limit formulas (15), (16) are valid up to $\xi_{20}^{-1} \leq 2\ell_{eff}^{-1}$ for $\xi_{10}^{-1} \leq 2\ell_{eff}^{-1}$. Figure 3 gives the ration of ξ_2/ξ_1 as a function of ξ_{20}^{-1} for a set of ℓ_{eff}/ξ_{10} . At $H \rightarrow 0$ the oscillation length ξ_2 goes to infinity. Therefore the ratio is diverges at $\xi_{20}^{-1} \rightarrow 0$. With H increase the ratio rapidly decreases approaching the law $\xi_2/\xi_1 \propto \xi_{20}^{-1}$ at $\xi_{20}^{-1} \geq 2$.

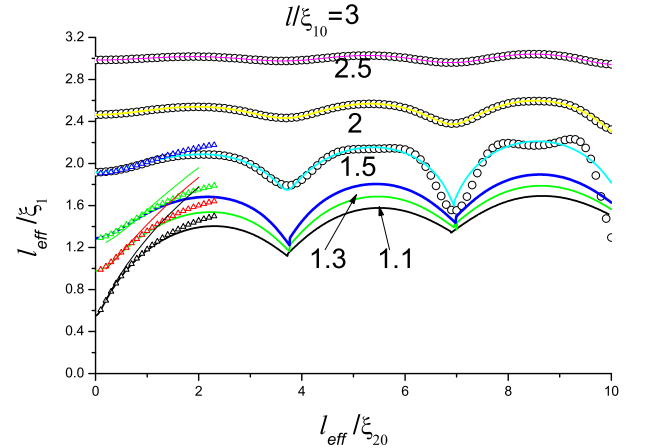


FIG. 1: The decay length ξ_1^{-1} as a function of ξ_{20}^{-1} calculated for different values of ξ_{10}^{-1} . The open circles are the asymptotic curves, which have been calculated from (20) for $\xi_{10}^{-1} = 2, 2.5$ and 3 . The open triangles are the asymptotic curves calculated from (15) for $\xi_{10}^{-1} = 1.1, 1.3, 1.5$ and 2 . The thin solid lines are the asymptotic dependencies following from Eq. (15) without the correction in the square brackets. These curves are calculated for $\xi_{10}^{-1} = 1.1, 1.3$ and 1.5 .

The discovered behavior of ξ_2 and ξ_1 is quite general and must be also observed in structures without ferromagnetic ordering. An example is a normal filament of finite length, which is placed between superconducting banks and is biased by a dc supercurrent. It was shown [28], that the minigap induced to this filament from the S electrodes is not a monotonous function of phase dif-

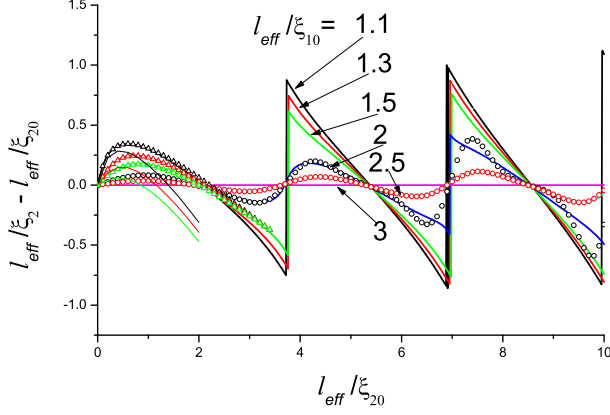


FIG. 2: The difference between decay length ξ_2^{-1} and ξ_{20}^{-1} as a function of ξ_{20}^{-1} calculated for different values of ξ_{10}^{-1} shown in figure. The open circles are the asymptotic curves, which have been calculated from (21) for $\xi_{10}^{-1}=2, 2.5$ and 3. The open triangles are the asymptotic curves calculated from (16) for $\xi_{10}^{-1}=1.1, 1.3, 1.5$ and 2. The thin solid lines are the asymptotic dependencies following from Eq. (16) without the correction in the square brackets. These curves are calculated for $\xi_{10}^{-1}=1.1, 1.3$ and 1.5.

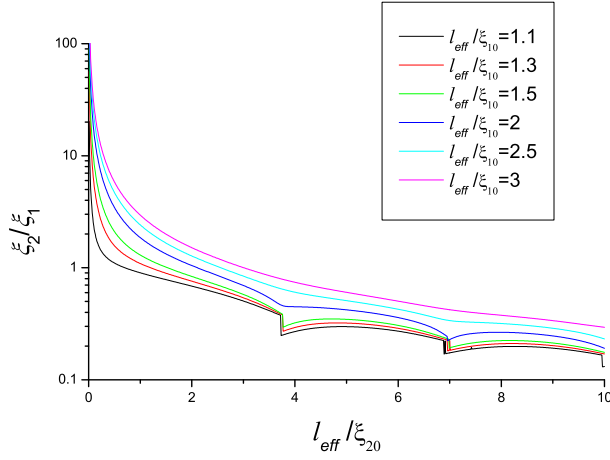


FIG. 3: The ratio of oscillation and decay lengths ξ_2/ξ_1 as a function of ξ_{20}^{-1} calculated for different values of ξ_{10}^{-1} .

ference across the structure. This behavior could be also explained in terms of specific dependencies of ξ_2 and ξ_1 upon electron mean free path in current biased systems.

In summary, by solving the linearized Eilenberger equations we have calculated the real, ξ_1 , and the imaginary, ξ_2 , part of a decay length as a function of exchange energy H and the mean free paths ℓ , ℓ_{so} , ℓ_z and ℓ_x for ordinary, spin-orbit and spin-flip electronic scattering in a ferromagnet. These parameters, ξ_1 , and ξ_2 , characterize penetration of superconducting correlations into a ferromagnet due to proximity effect and determine the decay and oscillation lengths of a critical current in long SFS Josephson structures. We have found the range of validity of the expressions (1), (2), which are commonly used for interpretation of experimental data. In particular, the dirty limit expressions (1) are valid if $\xi_{20}^{-1} \leq 0.5\ell_{eff}^{-1}$ for $\xi_{10}^{-1} \leq 0.5\ell_{eff}^{-1}$. The corrected expressions (15), (16) can be used in a broader range of $\xi_{20}^{-1} \leq 2\ell_{eff}^{-1}$ and $\xi_{10}^{-1} \leq 2\ell_{eff}^{-1}$. Further increase of exchange field makes the length ξ_2 smaller than ℓ_{eff} thus breaking down the validity of approximations used in derivation of the Usadel equations. It is interesting to note that in certain parameter range the jumps occur in the dependence of ξ_2 vs ξ_{20} , while ξ_1 remains a continuous function of ξ_{20} .

We have also demonstrated that the intuitive knowledge about the relation between ξ_1 and ξ_2 , based on the dirty limit theory, has very limited range of applicability and can not be used for $\xi_H > 5\ell$ or for $H\tau > 0.1$. In particular, an increase of H is not always accompanied by a decrease of ξ_1 and in a certain parameter range ξ_1 may even increase with H . The fact that one may combine reasonably large decay length with the smaller period of oscillations looks rather attractive for possible applications of SFS Josephson junctions.

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